THEORIES OF COSMIC STRUCTURE FORMATION

Major Questions

1. Is gravitational instability the driving mechanism?
2. What was the origin of primordial fluctuations?
3. What is the dark matter?
4. What are the values of cosmological parameters? (Ω_M, Ω_Λ, H_0, Ω_b, others)
5. What additional physics is important in galaxy formation?
6. What is the relation between observable tracers of structure and the underlying mass distribution?

Can't use observations to address $1-4$ without addressing 6, hence 5.

The Inflation plus Cold Dark Matter (CDM) Scenario

1. Yes.
2. Quantum fluctuations during inflation $\Longrightarrow$ Gaussian, adiabatic. (Alternatives: Topological defects. Isocurvature and/or non-Gaussian fluctuations from more complicated inflation models.)
3. Weakly interacting massive particles with negligible thermal pressure. (Alternatives: Hot dark matter. Baryons only. Non-GR gravity.)
4. Various possibilities, e.g.:
   - "Standard": $\Omega_M = 1$, $\Omega_\Lambda = 0$
   - "Mixed": $\Omega_{\rm CDM} \sim 0.8$, $\Omega_\nu = 1 - \Omega_{\rm CDM}$, $\Omega_\Lambda = 0$
   - "Open": $\Omega_M \sim 0.4$, $\Omega_\Lambda = 0$
   - "Lambda": $\Omega_M \sim 0.4$, $\Omega_\Lambda = 1 - \Omega_M$
   - "Ugly": $\Omega_{\rm CDM} = 0.3$, $\Omega_\Lambda = 0.3$, $\Omega_\nu = 0.1$, $\Omega_b = 0.05$

Observational Testing Grounds

- CMB anisotropies
- Large scale structure (redshift surveys, clusters, weak lensing)
- Properties of galaxies at low and high redshift
- High redshift structure (especially Lyman-alpha forest)
- "Classical" tests: H_0 measurements, deceleration/acceleration measurements (supernovae), age of universe, lensing

Can test models well with high precision constraints from one area or with complementary low precision constraints from different areas.

The Linear Theory Power Spectrum of CDM Models

- Large scales: $P(k) \propto k^{n_p}$ with $n_p \approx 1$
- Turnover at $\lambda = 2\pi/k \sim ct_{\rm eq}$
- Asymptotes to $P(k) \propto k^{n_p - 4}$ at very small scales
- Effective index on cluster and galaxy scales is $-1 \gtrsim n \gtrsim -2$ (for $P(k) \propto k^n$)

Generating Initial Conditions for a Cosmological Simulation

To initialize a simulation with $N_p = N^3$ particles:

(1) Generate linear density field on an N^3 grid.

- Starting in $\mathbf{k}$-space, draw real and imaginary parts of $\tilde{\delta}_{\mathbf{k}}$ from independent Gaussian distributions with variance $P(k)/2$. FFT to get $\delta(\mathbf{x})$.
- Simulation can only represent initial $P(k)$ over range $k = k_f = 2\pi/L_{\rm box}$ to $k = k_{\rm Nyq} = k_f(N/2)$.
- Example: moa:/usr1/dhw/Source/Array/ranfield_cdm.f

(2) Create "uniform" particle distribution: Poisson, grid, or glass.

(3) Perturb particles and assign velocities using the Zel'dovich approximation (Lagrangian linear perturbation theory):

$$\Psi(\mathbf{q},t) \;\equiv\; \mathbf{x}(\mathbf{q},t) - \mathbf{q} \;=\; \mathbf{g}_0(\mathbf{q})\frac{D(t)}{D(t_0)}\left(\frac{3}{2}\Omega_M H_0^2 a_0\right)^{-1}$$

$$\mathbf{v} \;=\; a\dot{\Psi} \;=\; \frac{\dot{D}}{D}(a\Psi) \;\approx\; \Omega_M^{0.6} H(a\Psi)$$

$D(t) =$ growing mode growth rate of linear perturbations

$\mathbf{q} =$ comoving position in an unperturbed universe

$\mathbf{g}_0 =$ gravitational acceleration at $t = t_0$ (in linear theory)

For practical purposes, it is easiest to think of this from the point of view of the continuity equation, which in the linear regime implies

$$\vec{\nabla}\cdot\Psi(\mathbf{x}) = -\delta(\mathbf{x}).$$

Fourier transforming both sides of this equation yields:

$$-2\pi i\mathbf{k}\cdot\tilde{\Psi}_{\mathbf{k}} = -\tilde{\delta}_{\mathbf{k}}$$

and thus

$$\tilde{\Psi}_{\mathbf{k}} = \frac{i\mathbf{k}}{2\pi k^2}\tilde{\delta}_{\mathbf{k}}.$$

So one can FFT $\delta(\mathbf{x})$, multiply each Fourier mode by $i\mathbf{k}/2\pi k^2$, and inverse FFT to get Ψ.

- In principle, one can start the simulation at any redshift high enough that fluctuations are in the linear regime on the scales represented in the simulation. An approximate criterion is $\langle\delta^2\rangle^{1/2} \lesssim 0.5$ on the grid scale, or (more reliable) that the largest displacement of any particle is $|\Psi| < L_{\rm box}/N$. However, for any given application it is best to experiment with the choice of starting redshift rather than rely on a rule of thumb.

- Instead of the Zel'dovich approximation, one can instead use 2nd-order perturbation theory solution to suppress transients, start at later redshift (Scoccimarro, astro-ph/9711187).