Computing Our Universe 1998

Special Topic – Gravitational Lensing

Eric Linder

Gravitational lensing = gravitational effects on light propagation

- Isolated Mass
- Statistical Structure
- Background Spacetime
- Almost all observations via light propagation

  WYSIsn'tWYG?

Origin of Lensing

- Equivalence Principle $\Rightarrow$ curved spacetime
- First Test of General Relativity
- Newtonian Picture

$$\Delta p = F\,\Delta t = (GM/b^2)\,(p/v)\,(2b/v)$$

$$\text{Deflection Angle } \alpha = \Delta p/p = 2GM/bv^2$$

- Einstein result: $\alpha = 4GM/bc^2$

  $2\times$ Newton since curvature relative to rulers + rulers curved

- PostNewtonian (PPN) result: $\alpha = (\gamma + 1)\times$ Newton

- Fermat's Principle: weak field = index of refraction

$$\delta \int n\, ds = 0$$

- History

  1919 Eclipse Expedition

  1935 Einstein calculates star lensing star

  1937 Zwicky calculates galaxy lensing

  1960s Liebes, Feynman, Refsdal

  1979 First observed lensing system: Weymann, Walsh, Carswell

Lensing is Everywhere

- Light passing by Sun

$$b = R_\odot$$
$$\alpha = 1.75''$$

$\Rightarrow$ Sun can create multiple images of sources $D > 550$ AU

- Fresnel zone

Phase mixing by lensing (parallax lensing)

$$D > \pi l^2/\lambda \approx 1\,(M/10^{11}M_\odot)\,kpc(\text{radio})$$

Physics of Lensing

4 Components of Photon Four-momentum

- k^0 - frequency: energy shift
- k^1 - phase: time delay
- $\vec{k}_\perp$ - transverse momentum - deflection

- Geometric optics: $\lambda \ll M \ll D$

$$\lambda_{vis} \approx 10^{-4.5} \text{ cm}, \lambda_{radio} \approx 10 \text{ cm}$$

$$M \approx 10^5 (M/M_\odot) \text{ cm}, D_{gal} \approx 10^{22.5} \text{ cm}, H_0^{-1} \approx 10^{28} \text{ cm}$$

$\Rightarrow$ Lensing is **achromatic**

- Energy Shift: stationarity $\Rightarrow$ conservation $\Rightarrow$ no redshift

- Surface Brightness: Liouville's theorem $I_\nu \nu^{-3}$ invariant

  frequency unchanged $\Rightarrow I_\nu$ constant

  $\Rightarrow$ surface brightness $\int I_\nu d\nu$ conserved

- Polarization: geodesic parallel transport $\Rightarrow$ no polarization shift

Violations:

- Time Dependence

- Differential Amplification

- Time delay: **geometric** and **potential** (index n) effects

- Deflection:

  - Single Ray Deflection
  - Ray Bundle Deviation (differential deflection)
  - Decomposition (**Ricci** and **Weyl**)

Convergence: matter within beam, including background spacetime

- changes solid angle of image vs. source
- Surface brightness unchanged $\Rightarrow$ flux $\sim$ solid angle

  $\Rightarrow$ Lensing causes **Amplification**

Shear: matter outside beam

- Changes image shape, can multiply image (**Macrolensing**)

Transform from source to image plane, mapping $R^2 \rightarrow R^2$

- Folding balloon inside glass sphere
- Catastrophe theory (**Caustics**)
- Odd image theorem

IMAGES

- Geometric constraint: light reaches observer
- Physical constraint: light obeys gravity

- Intersection $\longrightarrow$ Image Position
- Slope $\longrightarrow$ Amplification
- Area $\longrightarrow$ Time Delay

Lensing: What Is It Good For?

- Background Cosmology
  - Cosmological Nature of Redshift (z measures geometry)
  - H_0 (time delays, distances)
  - Cosmological Constant Constraints (lensing probabilities)
  - Testing Gravitation (e.g. disproof of MOND)
- Source Properties
  - Cosmic Microwave Background ($l > 5000$)
  - Time Dependent Sources (e.g. supernovae)
  - Amplification Bias (e.g. quasar-galaxy associations)
- Lens Properties
  - Proper Motions
  - Numbers, Masses, and Structure
  - Microlensing
  - Gravitational Waves

Lensing Probability

Optical Depth: $\tau = \sigma \, n \, dl$

- σ – cross section: point mass $A \sim b^{-1}$, $\sigma(> A) = \pi b^2$
- n – number density of lensing objects, $\Omega_l(M)$
- dl – path length element, cosmology dependent (Λ)

Probability $p(A) \approx \Omega_l[H_0/\bar{H}(z)] \, A^{-3}$,

Amplification Bias

$$N(F) = \int dA\, p(A)\, N_o(F/A)$$

- Faint end: $N_o(f) \sim F^{-\alpha}$

$$N(F) = \int_1^\infty dA\, A^{-3} (F/A)^{-\alpha} \sim F^{-\alpha}$$

- Bright end: $N_o(F) \sim F^{-\beta}$

$$\begin{aligned} N(F) &= \int_1^{F/F_*} dA\, A^{-3} (F/A)^{-\beta} + \int_{F/F_*}^\infty dA\, A^{-3} (F/A)^{-\alpha} \\ &\sim F^{-\beta} F^{-2+\beta} \sim F^{-2} \end{aligned}$$

Dark Matter: MACHOs

- LMC, SMC (galactic halo) ; Bulge
- 50% of halo is $0.5\,M_\odot$ dark matter (old white dwarfs?)
- $< 20\%$ of halo is in $10^{-6} - 10^{-2} M_\odot$ objects

Computational Gravitational Lensing

- Lens Reconstruction
- Ray Shooting – Statistical Lensing
- Ray Shooting – Cosmological Model and FRWness