Homework Exercise 2
Integration/Interpolation: Cosmological Ages/Magnitudes
22 June – due 24 June

General Hints:

- Label plots appropriately, including axes, units, curve parameters, and other plot parameters.
- Always carry out analytic checks and exhibit these formulas.
- Think about the physical meaning of the results – do the numbers obtained seem reasonable? Comment on the physical implications of results, e.g. influence of cosmological constant or curvature.
- Understand the point and meaning of the numerical procedures – don't apply them blindly.

1. Write a program to perform Romberg Extrapolation of Simpson's Rule.

2. Apply the program to calculation of the cosmological lookback time-redshift relation for a model with arbitrary regular matter (dust) density Ω_m and cosmological constant Ω_Λ:

$$H_0 t(z) = \int_1^{1+z} dy\, y^{-1}[\Omega_m y^3 + (1 - \Omega_m - \Omega_\Lambda)y^2 + \Omega_\Lambda]^{-1/2}$$

for the models $(\Omega_m, \Omega_\Lambda)$=(0.4,0), (0.4,0.4), (0.4,0.6).
Hint: test your program against the analytic result for the (1,0) model.

3. Use the results to make tables of the values of redshift vs. time over the range $z = 0 - 5$. Write programs using both linear and spline interpolation on the tables to return the value of lookback time given the redshift (see the handouts on how to solve the tridiagonal system of linear equations for the spline fits). Plot the table values as points and the spline fits as curves for each of the models. Investigate and consider what different size bins are needed for the linear and spline fits.
Hint: use your knowledge of the physical meaning of $t(z)$ to choose appropriate boundary conditions for the spline.

4. Integrate to infinity to get the age of the universe for each of the models.
Hint: consider how to treat properly the upper limit at infinity.

$\star$5. The most current measurements of the energy density of the universe, including the cosmological constant, use high redshift supernovae to plot magnitude versus redshift (the **Hubble diagram**). If they are standard candles, their observed magnitude is related to the luminosity distance by $m(z) \sim 5\log[H_0 r_l(z)]$, where

$$H_0 r_l(z) = (1+z)(1-\Omega)^{-1/2}\sinh\left[(1-\Omega)^{1/2}\int_1^{1+z} dy\,[\Omega_m y^3 + \Omega_\Lambda + (1-\Omega)y^2]^{-1/2}\right],$$

for universes with matter plus cosmological constant, $\Omega = \Omega_m + \Omega_\Lambda$.

a) Plot the Hubble diagram m vs. z for the models $(\Omega_m, \Omega_\Lambda) = (1, 0)$, $(0.4,\ 0)$, $(0.2,\ 0)$, $(0.25,\ 0.75)$, $(0.4,\ 0.6)$ out to $z = 2$.

b) To see the differences between models better, plot the deviation $m(z) - m_f(z)$ from a fiducial model $(0.2,\ 0)$. Discuss the ability to determine the correct model if observations have an accuracy of 0.2 magnitudes.