Homework VI: N-Body

1.) Write a monopole Oct-tree code to calculate accelerations and potential energies. Use the standard geometrical opening criterion, θ_{max}. Also use Plummer softening. For Plummer softening the potential energy is

$$\phi_i = -\sum_{j,j\neq i} \frac{Gm_j}{\sqrt{|\mathbf{r}_{ij}|^2 + \epsilon^2}}$$

For simplicity assume that $G = 1$.

2.) Show how the acceleration errors (total, radial, and tangential) change with θ_{max}. Show this using a King model realized with 32768 particles. Assume $\epsilon = 0.2$. Files containing King models are in the directory /hosts/A/home/trq/hw6 in files called king*number*. For example this file is king32768. The King models have $W_0 = 7$ and a core radius of 1.0. The file has TIPSY ascii format. The file king32768.acc has the correct accelerations calculated using direct summation. The file has a TIPSY ascii header followed by all the x accelerations then all the y accelerations then all the z accelerations. What θ_{max} gives median errors with less than 0.5%?

3.) Using the Linux **time** command and King models realized with varying numbers of particles, show that the acceleration calculation scales as $O(n \ln n)$. For what particle number does the tree code out-perform a direct summation code for a θ_{max} that gives median errors less than 0.5%?

4.) Use the tree acceleration code to write a leapfrog N-body code (all particles having the same timestep). Apply this code to king32768 and demonstrate that it is in equilibrium.

Hint: to choose a starting point for the proper timestep notice that the maximum density is one.

5.) Try varying the timestep, gravitational softening length, and particle number to determine their effect on maintaining equilibrium.

6.) Try implementing quadrupole expansions for the nodes and/or spline softening and repeating parts 2–5.