Computing Our Universe

Cosmology IV – Cosmological Model

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1 COSMOLOGICAL MODEL

A cosmological model consists of three parts:

- **Spacetime** global structure, e.g. homogeneity and isotropy
- **Theory of gravitation**, e.g. general relativity
- **Ingredients** – amounts and eqn of state, e.g. matter, radiation, etc.

Observations make the first two good bets.

Third is commonly parametrized with three constants: H_0, Ω, q_0.

Why these? – exactly those comprising Friedmann equations.

$$H_0^2 = H_0^2\Omega - k \quad \longrightarrow \quad k = (\Omega - 1)\, H_0^2$$
$$q_0 = \frac{1}{2}\sum(1+3w)\Omega_w \quad \longrightarrow \quad q_0 = \frac{1}{2}\Omega + \frac{3}{2}\sum w\Omega_w$$

Also from Taylor expanding the expansion factor about the present.

$$\begin{aligned} a(t) &= a_0 + \dot{a}_0(t-t_0) + \frac{1}{2}\ddot{a}_0(t-t_0)^2 + \ldots \\ &= a_0\left[1 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2 + \ldots\right] \end{aligned}$$

Clearly shows *insufficiency* of these 3 parameters to specify cosmology.

Need infinite derivatives, or $a(t)$ itself.

Good choice is H_0 plus full set of ingredients Ω_w.

2 DISTANCES

2.1 Comoving and Proper Distances

To determine the cosmological parameters one must make observations of the spacetime geometry, most commonly as distances and related quantities as a function of redshift.

Distinguish between **proper distances** and **comoving distances**. Related over infinitesimal intervals (from metric) by

$$dr_p = a(t)\, dr_c = (1+z)^{-1} dr_c\,.$$

Consider a simulation volume of comoving size L:

- Today ($z = 0$)

  Comoving size L

  Proper size L (by definition proper = comoving today)

- Redshift z

  Comoving size L (constant with redshift)

  Proper size $L/(1+z)$

Example: Velocity width of simulation box at z

Hubble's law: $\Delta v = H(z)\Delta r_c = H(z)\, L$

Example: Angular width of simulation box at z

Must work either in all comoving or all proper distances - *not* a mixture:

Incorrect: $\theta = L/r_a(z)$.

All proper: $\theta = \frac{L/(1+z)}{r_a(z)}$.

All comoving: $\theta = \frac{L}{(1+z)r_a(z)}$.

Of course same answer either way.

The proper and comoving distances are given by the metric,

$$r_p = \int dr_p = \int_{t(z)}^{t_0} dt = -\int_0^z dz\, \frac{da}{dz}\frac{dt}{da} = \int_0^z dz\,(1+z)^{-1}H^{-1}(z)$$

$$\int_0^{r_c} dr\,(1-kr^2)^{-1/2} = \int_{t(r_c)}^{t_0} dt\, a^{-1} = \int_0^z dz\, H^{-1}(z)$$

$$H_0\, r_c(z) = (1-\Omega)^{-1/2}\sinh\left[(1-\Omega)^{1/2}\int_0^z dz\;(H_0/H(z))\right]$$

2.2 Observables and Distances

Unlike in Euclidean space, in curved spacetime there are different types of (related) distances for different quantities observed.

Figure 3.3: Euclidean vs. curved distances

Most commonly used in cosmology are the **angular size distance** and **luminosity distance**. They are related to comoving distance by

$$(1+z)\, r_a = r_c = (1+z)^{-1} r_l.$$

H_0 sets overall scale (being only dimensional parameter).

Uncertain of exact value so write $H_0 = 100h\,\mathrm{km\,s^{-1}Mpc^{-1}}$.

Often measure distances in h^{-1}Mpc.

For low redshifts, at $\mathcal{O}(z^2)$ only the cosmological model parameter q_0 enters; rest of the model enters only at $\mathcal{O}(z^3)$ for any distance measure r:

$$H_0\, r(z) = z + f(q_0)\, z^2 + g(\Omega_w)\, z^3 + \ldots$$

Low redshift observations can never determine the cosmological model.

Holds as well for observables derived from distances, e.g.

- **number-redshift** relations (from volume elements $dV = r_a^2 dr_p$)
- **magnitude-redshift** relations ($m \sim 5 \log r_l$)
- **angular size-redshift** relations ($\theta \sim 1/r_a$)

But at high redshifts there are other problems.

3 COSMOLOGICAL TESTS

3.1 Evolution

Observations are harder at higher redshifts. But other, more insidious problem – **evolution**. E.g. magnitude of object depends not only on its distance (and hence cosmological model for a measured redshift) but its intrinsic luminosity. Since this may change with age, i.e. redshift, then the properties of the individual object or class of objects and those of the cosmology are *entangled*.

Example: Luminosity Evolution and q_0

If the test objects' luminosities evolve with time then

$$
\begin{aligned}
L(t) &= L(t_0) + (t_0 - t)\dot{L}_0 + \ldots \\
&\approx L_0 \, (1 - zH_0^{-1}\dot{L}_0/L_0) \\
&\approx L_0 \, (1 - zH_0^{-1}/\tau)
\end{aligned}
$$

with τ the timescale of evolution. If we derive q_0 from the magnitude or apparent luminosity

$$l = \frac{L(z)}{4\pi r_l^2} \approx \frac{L(z)H_0^2}{4\pi z^2}[1 + z(q_0 - 1)]$$

then ignorance of the evolution would lead to a bogus determination,

$$q_0(\text{obs}) = q_0(\text{true}) - H_0^{-1}/\tau.$$

Evolution timescale could well be less than or of order the Hubble time, $\tau \leq H_0^{-1}$, so relative error enormous.

Ignorance of evolution could lead us to think q_0 is smaller than actual, e.g. the sign of a large cosmological constant. See just this possibility of error in the current best cosmological model determinant, Type Ia supernovae.

3.2 Supernovae and Cosmic Microwave Background

Problems:

- lack of leverage of low redshift observations
- difficulty of high redshift observations
- problem of evolution in a general class of objects

Current best bets for determining the cosmological model are

- angular size distance test on certain perturbations to CMB
- magnitude test on Type Ia supernovae as standard candles

with a less developed, outside possibility being

- number test on a class of gravitationally lensed objects

FIGURE: Supernovae $\Omega - \Lambda$

FIGURE: Supernovae $\Omega - \Lambda$ with Evolution

FIGURE: CMB Ω Animations

3.3 Mimic Models

Key parameter in many cosmological tests is $H(z)$.

One ingredient can mimic another over small redshift range if contribute same to $\sum_w$ in $H(z)$ or other key parameter:

$\Rightarrow$ **cosmological parameter degeneracy**

Different redshifts (and tests) more sensitive to different combinations.

- At $z \approx 0.5$ (SN test), roughly sense $\Omega - \Lambda$
- At $z \approx 3$ (galaxy power spectrum), roughly sense Ω
- At $z \approx 1000$ (CMB test), roughly sense $\Omega + \Lambda$

Correlating improves parameter estimation thru **complementarity**.

FIGURE: Supernovae + CMB Complementarity

3.4 Other constraints

Cosmological ingredients also probed via

- direct detection: laboratory dark matter searches
- theory: nucleosynthesis abundances
- simulation: large scale structure formation

Always must understand underlying physics (cf. evolution).

Confidence in answer cannot exceed knowledge of astrophysics involved!