*Computing Our Universe*

**Cosmology III – Evolution and Fate of the Universe**

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# 1 REDSHIFT

## 1.1 Frequency of Light

Four arguments for the redshift (decrease in frequency) of light:

- Scale factor $a(t)$ multiplies all free distances, so wavelengths should expand (*cf.* adiabatic box in quantum theory or organ pipe)
- Radiation density $\rho_\gamma \sim a^{-4}$ so $E_\gamma \sim \nu \sim a^{-1} \sim 1+z$
- Equivalence Principle in nonstationary spacetime:
  different times $\Rightarrow$ different geometries $\Rightarrow$ different accelerations
  $\to$ pseudoDoppler shift
- Null geodesic equation $ds = 0 \Rightarrow dt/a = F(r)\,dr$

$$\int_{t_e+\Delta t_e}^{t_o+\Delta t_o} dt\, a^{-1} = \int_0^r dr\, F(r) = \int_{t_e}^{t_o} dt\, a^{-1}$$
$$\int_{t_e}^{t_e+\Delta t_e} dt\, a^{-1} = \int_{t_o}^{t_o+\Delta t_o} dt\, a^{-1}$$

Adiabatic approximation says $a(t)$ changes little over wave period, so

$$\frac{\Delta t_e}{a_e} = \frac{\Delta t_o}{a_o}$$
$$z \equiv \frac{\lambda_o - \lambda_e}{\lambda_e} \equiv \Delta\lambda/\lambda = \frac{a_o}{a_e} - 1$$

### 1.2 Observations of redshift

- Astronomical redshifts first observed 1912. Today $10^5$ observed, $\approx 10$ negative. SDSS will find $10^6$.

- 4 properties: Positivity ($\lambda_o > \lambda_e$), Frequency Independence, Linearity ($z \sim r$), Isotropy
- Explained only by **expansion of universe**.

Frequency independence from geometric optics ($\lambda \ll H_0^{-1}$). Isotropy from isotropic universe. Doppler shift would need sources receding (*expansion*) for positivity, and coasting (*e.g. after explosion*) for linearity – leads back to Big Bang picture. Doppler shifts do exist though at level $z < \mathcal{O}(10^{-3})$.

### 1.3 Scaling

Other physical quantities scale with wavelength

$$\rho_\gamma \sim a^{-4} \qquad\qquad T_\gamma \sim a^{-1}$$

Expansion leads to lower density, lower temperature (analogy to air escaping thru lips).

> *Observation of expansion leads to picture of hot dense early universe:* **Big Bang model** *with age of order* $H_0^{-1}$

## 2 HUBBLE EXPANSION

Expansion of universe implicit in general relativity, recognized in 1916. Linearity of redshift predicted by Weyl in 1923 as consequence of Cosmological Principle. Observations confirmed by Hubble in 1927.

$$v \equiv \frac{dr}{dt} = \frac{d(ar_0)}{dt} = r_0 \dot{a} = \frac{\dot{a}}{a} r \equiv Hr$$

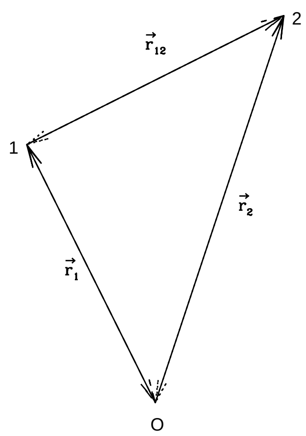
Uniform expansion implies linear relation called **Hubble's Law**

$$\vec{v} = H_0 \vec{r}$$

Expansion is predicted, not put in by hand.

Homogeneity requires vector linearity – no effect if offset by constant vector:

$$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1 = H_0(\vec{r}_2 - \vec{r}_1) = H_0 \vec{r}_{21}$$



Could imagine more general tensor law: $\vec{v} = \mathbf{H}\vec{r}$ with Hubble matrix $\mathbf{H}$ composed of

- uniform expansion $H\delta_{ij}$ (diagonal)
- shear $\sigma_{ij}$ (symmetric tracefree)
- rotation $\omega_{ij}$ (antisymmetric)

Stringent CMB limits to $\mathcal{O}(10^{-6})$.

**Hubble flow**: *radial* velocity from expansion.

**Proper motion**: transverse to line of sight (nonexpansion).

**Peculiar velocity**: total nonexpansion motion; radial part gives true Doppler shift, $< \mathcal{O}(10^{-3})$.

Linear relation is just first derivative term in Taylor expansion of $a(t_e) = a(t_o) - (t_o - t_e)\,\dot{a}(t_e) + \ldots$. Breaks down at $z \geq 1$ because of spacetime curvature ($H(z)$, no unique $r$) – important probe of cosmological model, e.g. high redshift supernovae test.

## 3 FATE OF THE UNIVERSE

### 3.1 Curvature

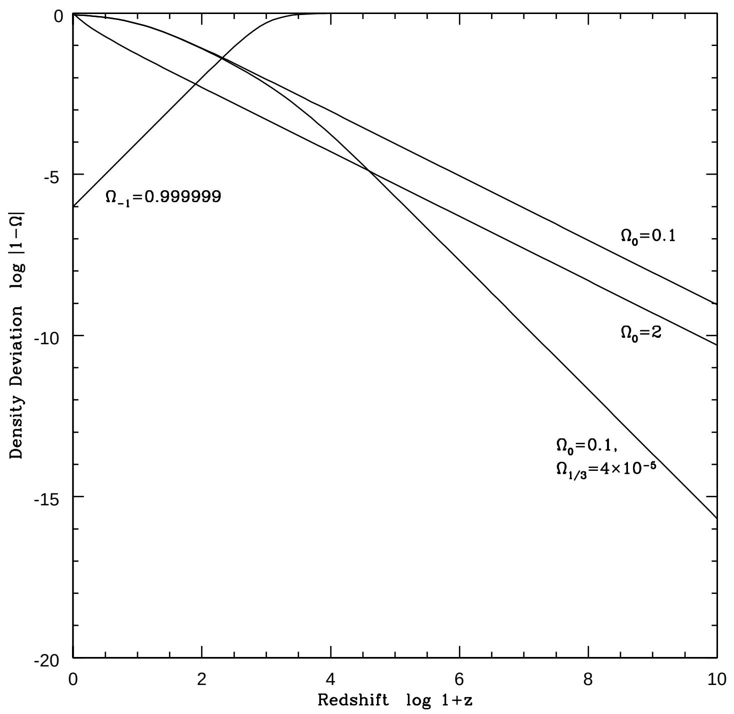
Velocity Friedmann eq relates expansion rate $H$ to matter content

$$H(z) \equiv (\dot{a}/a) = H_0 \left[ \sum \Omega_w (1+z)^{3(1+w)} + (1-\Omega)(1+z)^2 \right]^{1/2}$$

Can rewrite in terms of curvature constant

$$k = [\Omega(z) - 1] a^2 H^2(z) = (\Omega - 1) a_0^2 H_0^2$$

Different components cause curvature density $1 - \Omega$ to evolve differently: if $w > -1/3$ (normal) then $|1 - \Omega|$ increases with time, if $w < -1/3$ (exotic) then $|1 - \Omega|$ decreases with time. Fate of universe controlled by future curvature.



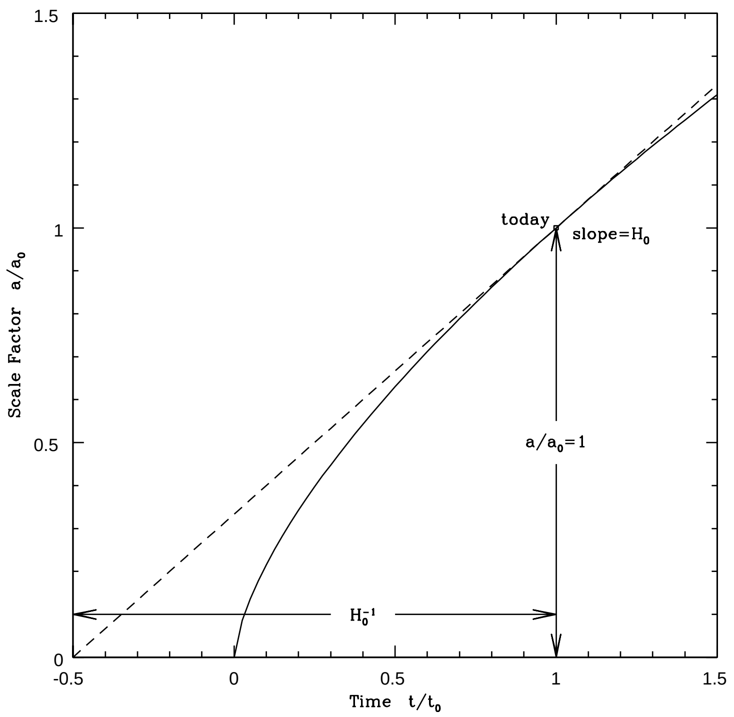
Today, after many characteristic Hubble expansion times, universe has $1 - \Omega \approx 0 - 0.75$, close to unstable critical density $\Omega = 1$. Analogous to pencil standing on point. Suggestive of exotic matter episode driving $|1 - \Omega| \to 0$ (inflation).

### 3.2 Deceleration

Acceleration Friedmann equation gives evolution of expansion rate, hence ultimate fate. Sensitive to EOS. Define **deceleration parameter**

$$q_0 = -\frac{\ddot{a}a}{\dot{a}^2}$$

Gives *spacetime* curvature.

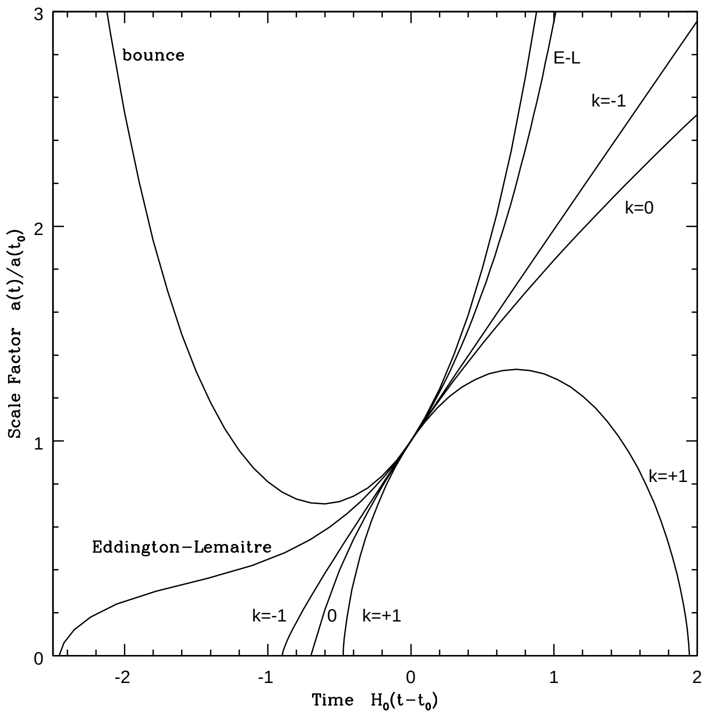


### 3.3 Asymptotic Future

Spatial curvature can be negative, zero, or positive. For normal EOS (*strong energy condition:* $w > -1/3$) this gives three fates:

**Fates of $w > -1/3$ Universes**

| Curvature | | Density | Fate |
|---|---|---|---|
| $k < 0$ | Negative curvature, | $\Omega < 1$ | Open universe $\rightarrow$ Big Chill |
| $k = 0$ | Flat or zero curvature, | $\Omega = 1$ | Critical universe $\rightarrow$ Big Coast |
| $k > 0$ | Positive curvature, | $\Omega > 1$ | Closed universe $\rightarrow$ Big Crunch |




To determine fate, need to measure densities and EOS of components of universe (matter, cosmological constant, etc.). Search for $\Omega$, $H_0$, $\Omega_w$.

**Figure: Part 1/2 summary**