Computing Our Universe

Cosmology II – Handout

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Review of Principles

Equivalence Principle
$\Rightarrow$ Metric Description

Homogeneity and Isotropy
$\Rightarrow$ Metric is Robertson-Walker
$\Rightarrow$ Energy-Momentum Tensor is in perfect fluid form

Gravitational Field Equations (general relativity)
& Homogeneity and Isotropy
$\Rightarrow$ Friedmann Equations for evolution of spacetime

Equation of State
& Friedmann Equations
$\Rightarrow$ Evolution of matter

Friedmann Equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p)$$
$$(\dot{a}/a)^2 = \frac{8\pi}{3}\rho - ka^{-2}$$
$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$

Equation of State

$$p = w\rho$$

Density Evolution

$$\rho_w \sim a^{-3(1+w)}$$

Time Evolution

$$a \sim t^{2/[3(1+w)]}$$

Components' EOS and Evolution

Radiation	$\rho_{1/3} \sim a^{-4}$	$a \sim t^{1/2}$	$t_0 = (1/2)H_0^{-1}$
Nonrelativistic matter	$\rho_0 \sim a^{-3}$	$a \sim t^{2/3}$	$t_0 = (2/3)H_0^{-1}$
Curvature	$\rho_{-1/3} \sim a^{-2}$	$a \sim t$	$t_0 = H_0^{-1}$
Cosmological constant	$\rho_{-1} \sim a^0$	$a \sim e^{H_0 t}$	$t_0 = \infty$