Computing Our Universe

Cosmology I – Handout

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Vital Statistics of Our Universe

$R \approx 10^{28}\,\mathrm{cm}$	$M \approx 10^{22}\,M_\odot$
$N_b \approx 10^{79}$	$\rho \approx 10^{-29}\,\mathrm{g\ cm^{-3}}$
$T \approx 3\,\mathrm{K}$	$t \approx 10^{10}\,\mathrm{y}$
$s \approx 10^{10}k$	$p \approx 10^{-19}\,\mathrm{atm}$

$$\text{Energy} \approx 0 \approx \text{Charge}$$

Given are a characteristic size of the universe R, total mass M, number of baryons N_b, mass density ρ, basic temperature T, age t, entropy per baryon s, and pressure p.

Astrophysical Units

Length:

- 1 Megaparsec (Mpc) $= 10^6\,\mathrm{pc} \approx 3.09 \times 10^{24}\,\mathrm{cm}$
- 1 Hubble length $(H_0^{-1}) \approx 3000h^{-1}\,\mathrm{Mpc} \approx (0.93h^{-1}\times)10^{28}\,\mathrm{cm}$
- Galaxy size $\approx 30\,\mathrm{kpc}$
- Cluster size $\approx 3\,\mathrm{Mpc}$

Mass:

- 1 solar mass $(M_\odot) \approx 1.99 \times 10^{33}\,\mathrm{g}$
- 1 proton mass $(m_p) \approx 1.67 \times 10^{-24}\,\mathrm{g}$
- Galaxy mass $\approx 10^{12} M_\odot$
- Cluster mass $\approx 10^{14} M_\odot$
- Galaxy overdensity $\delta\rho/\rho \approx 10^5$
- Cluster overdensity $\delta\rho/\rho \approx 10$

Time:

- 1 year (y) $\approx 3.16 \times 10^7\,\mathrm{s} \approx 10^{7.5}\,\mathrm{s}$
- 1 Hubble time $(H_0^{-1}) \approx (0.98\times)10^{10}h^{-1}\,\mathrm{y} \approx 3h^{-1} \times 10^{17}\,\mathrm{s}$

Geometric Units

Mass:	$[M] = [L]c^2/G$	$1\,M_\odot \approx (1.48\times)\,1\,\mathrm{km}$
Time:	$[t] = [L]/c$	$1\,\mathrm{y} = 1\,\mathrm{ly} \approx (0.95\times)10^{18}\,\mathrm{cm}$
Temperature:	$[T] = [E]/k = [L]c^4/Gk$	
	$300\,\mathrm{K} \approx (1/40)\,\mathrm{eV}$	$10^{10}\,\mathrm{K} \approx (0.86\times)\,1\,\mathrm{MeV}$

Forms of the Robertson-Walker Metric

Isotropic:

$$ds^2 = -dt^2 + a^2(t)\,(1 + kr^2/4)^{-2}[dr^2 + r^2 d\Omega^2]$$

Comoving:

$$ds^2 = -dt^2 + a^2(t)\,[dr^2 + \chi^2 d\Omega^2]$$

$$\chi = (\sin r,\ r,\ \sinh r) \quad \text{for} \quad k(>, =, <)0$$

Standard:

$$ds^2 = -dt^2 + a^2(t)\,[(1 - kr^2)^{-1} dr^2 + r^2 d\Omega^2]$$

Conformal:

$$ds^2 = a^2(\eta)\,[-d\eta^2 + (1 + kr^2/4)^{-2}(dr^2 + r^2 d\Omega^2)]$$

Einstein Field Equations

(spacetime) $\quad G_{ab}[g_{ab}] \equiv R_{ab} - (1/2)Rg_{ab}$

(Einstein Field Eqs) $\quad R_{ab} - (1/2)Rg_{ab} = 8\pi T_{ab}$

(particle physics) $\quad 8\pi T_{ab} \equiv (2/\sqrt{-g})[\delta(\sqrt{-g}\mathcal{L})/\delta g^{ab}]$