*Computing Our Universe*

# Cosmology I – Whence Cosmology?

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# 1 OVERVIEW

## 1.1 Whence Cosmology?

How can we approach a scientific description of the entire universe? How can we substantiate the hubris of "Computing Our Universe"? By using fundamental principles underlying physics – test.

## 1.2 Vital Statistics of the Universe

**Table 1.1: Vital Statistics**

| | |
|---|---|
| $R \approx 10^{28}\,\mathrm{cm}$ | $M \approx 10^{22}\,M_{\odot}$ |
| $N_b \approx 10^{79}$ | $\rho \approx 10^{-29}\,\mathrm{g\ cm^{-3}}$ |
| $T \approx 3\,\mathrm{K}$ | $t \approx 10^{10}\,\mathrm{y}$ |
| $s \approx 10^{10} k$ | $p \approx 10^{-19}\,\mathrm{atm}$ |

$$\text{Energy} \approx 0 \approx \text{Charge}$$

$R$ = characteristic size of the universe

$M$ = mass within the corresponding volume

$N_b$ = number of baryons ("ordinary particles") in that volume

$\rho$ = average mass/energy density in universe

$T$ = characteristic background temperature of universe

$t$ = age of universe

$s$ = entropy per baryon of universe in units of Boltzmann's constant

$p$ = average pressure of universe (in units of Earth's atmosphere)

Energy and charge, both conserved quantities, close to zero. Suggests possibility universe could have arisen from the null state – the **vacuum** – through **quantum fluctuations**. High entropy ($s$ conserved).

These observed parameters give rough characterization of *our* universe. Maybe not fundamental. But is possible to describe our physical universe in fairly simple way, can treat quantitavely and scientifically.

## 1.3 Astrophysical Units

Great variety of scales and units enter the study of cosmology, from the subatomic to astronomical.

**Table 1.2: Astrophysical Scales**

| | |
|---|---|
| Length: | 1 Megaparsec (Mpc) $= 10^6\,\mathrm{pc} \approx 3.09 \times 10^{24}\,\mathrm{cm}$ |
| | 1 Hubble length $(H_0^{-1}) \approx 3000h^{-1}\,\mathrm{Mpc} \approx (0.93h^{-1}\times)10^{28}\,\mathrm{cm}$ |
| | Galaxy size $\approx 30\,\mathrm{kpc}$ |
| | Cluster size $\approx 3\,\mathrm{Mpc}$ |
| Mass: | 1 solar mass $(M_\odot) \approx 1.99 \times 10^{33}\,\mathrm{g}$ |
| | 1 proton mass $(m_p) \approx 1.67 \times 10^{-24}\,\mathrm{g}$ |
| | Galaxy mass $\approx 10^{12} M_\odot$ |
| | Cluster mass $\approx 10^{14} M_\odot$ |
| | Galaxy overdensity $\delta\rho/\rho \approx 10^5$ |
| | Cluster overdensity $\delta\rho/\rho \approx 10$ |
| Time: | 1 year (y) $\approx 3.16 \times 10^7\,\mathrm{s} \approx 10^{7.5}\,\mathrm{s}$ |
| | 1 Hubble time $(H_0^{-1}) \approx (0.98\times)10^{10}h^{-1}\,\mathrm{y} \approx 3h^{-1} \times 10^{17}\,\mathrm{s}$ |

Note Hubble length and time has a fiddle factor $h$, thought to lie between 0.5-1 with a best guess around 0.75.

## 2 EQUIVALENCE PRINCIPLE

How describe physics itself of the universe?

Einstein said laws of physics, i.e. *form* of physics, same everywhere in universe – **Principle of Equivalence**.

### 2.1 Principle of Equivalence

Theoretical form:

> *One can choose a coordinate system such that locally the form of the laws of physics is that in special relativity.*

Thus only need to understand special relativity and coordinate transformations to write laws of physics in form valid anywhere in universe.

Experimental form:

$$m_i/m_g = \text{constant}$$

Acceleration caused by forces (inertial mass $m_i$) is equivalent to properties of gravitation (gravitational mass $m_g$) – independent of internal properties of objects. Verified to $\mathcal{O}(10^{-12})$.

Motto:

$$\text{Acceleration} = \text{Gravitation} = \text{Curvature}$$

Example: physics of bending of light independent of interpretation used; whether the local coordinate system is thought to be experiencing acceleration, gravitational force, or curved geometry.

**FIGURE 1.3: triple view of bending of light**

## 2.2 Metric

Mathematically, Principle of Equivalence implies that "distance" **interval** $ds$ between events in spacetime must be given by a quadratic form of the coordinates $dx^a$:

$$ds^2 = g_{ab} dx^a dx^b$$

All physics contained in quadratic form or **metric** $g_{ab}$ which gives the relation between distances and coordinates, measuring the **curvature** of the spacetime.

The curvature can take several forms, e.g. smoothly convex (positive), smoothly concave (negative), smoothly flat (zero), or wrinkly.

**FIGURE 1.4: curvature on a sweater**

Warp and weft threads form coordinate system. Metric gives

- distances: Pythagorean theorem $\rightarrow$ nonEuclidean geometry
- areas (volumes)
- angles (e.g. quadrilateral b has $> 360°$)
- curvature (positive, negative, flat, smooth, wrinkly)

General relativity is math and physics of $g_{ab}$ – need to know almost none for our universe.

# 3 COSMOLOGICAL PRINCIPLE

How interpret whole universe from local observations?

## 3.1 Isotropy

Observations indicate universe looks much the same in all directions – isotropy. E.g. counting galaxies on the sky. Much more accurately by measuring temperature of cosmic background radiation field – uniform in direction to $\mathcal{O}(10^{-5})$.

### 3.2 Homogeneity

Is universe much the same not just in all directions on the sky, but at all points in space – homogeneity. Want 2-d sky information $\rightarrow$ 3-d map of the universe.

Only one local vantage point so isotropy about that one point doesn't guarantee homogeneity. But would if that one point were random location.

**Copernican Principle** says we not in preferred location at center of universe (Cosmological Principle or Principle of Cosmic Modesty). If accepted then our universe can be taken to be homogeneous.

Seems reasonable we not in special location, but want to test. Must build 3-d map through astronomical surveys.

### 3.3 Evolution

*But* finite speed of light says looking out in distance is looking back in time – things may change.

Can't directly compare distant objects to nearby ones without understanding **evolution**. To verify Cosmological Principle – to do cosmology – must also study astrophysics and understand galaxies and clusters.

Through combination of *assuming* Cosmological Principle and using partial and ever improving verifications of it from large scale surveys, we will adopt homogeneity as a property of our universe.

### 3.4 Robertson-Walker Metric

Isotropy and homogeneity immediately give specific form to the metric $g_{ab}$ which determines the spacetime geometry and physics of the universe.

This is **Robertson-Walker** form. Distance interval $ds$ is simple; in some common coordinate choices:

**Table 3.1: Forms of the Robertson-Walker Metric**

Isotropic:

$$ds^2 = -dt^2 + a^2(t)\,(1 + kr^2/4)^{-2}[dr^2 + r^2 d\omega^2]$$

Comoving:

$$ds^2 = -dt^2 + a^2(t)\,[dr^2 + \chi^2 d\omega^2]$$

$$\chi = (\sin r,\ r,\ \sinh r) \quad \text{for} \quad k(>, =, <)0$$

Standard:

$$ds^2 = -dt^2 + a^2(t)\,[(1 - kr^2)^{-1} dr^2 + r^2 d\omega^2]$$

Here $d\omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$ is the usual interval on the 2-sphere.

Two important properties to note about RW metric:

1) stays isotropic and homogeneous over time (never mixes $r$ and $t$ and only depends on $r$ as a whole, never $x, y, z$)

2) involves one variable dependent on time, $a(t)$, and one constant, $k$

## 3.5 Distances and Kinematics (curvature and expansion)

The constant $k$ enters the spatial part of the metric and causes it to deviate from the Euclidean form; $k$ is the **spatial curvature constant**.

- spatial curvature only constant because homogeneous, isotropic
- spatial curvature different from spacetime curvature;
  even if $k = 0$ there is spacetime curvature: 4-d not Euclidean

$dr$ is the **coordinate distance** interval.
$(1 - kr^2)^{-1/2} dr$ is the **comoving distance** interval.
$a(t)\,(1 - kr^2)^{-1/2} dr$ is the **proper distance** interval

The parameter $a(t)$ multiplies the spatial metric and is called the **scale factor**. Note $a(t)$ just scales proper to comoving distance intervals: $dr_{\rm prop} = a\,dr_{\rm com}$ – evolution in time scaled out in comoving frame so coordinate positions constant – objects "move with" the spacetime evolution and appear fixed in that frame. Later call $a(t)$ **expansion parameter**.

Often normalize $a(t)$ so $a(\text{today}) = 1$. Sometimes normalize $k = \pm 1$. Can't do both!

Can read off all kinematics of universe from metric – redshift, Hubble expansion.

For dynamics (forces), need to know $a(t)$ – need to relate metric to matter content through theory of gravitation, e.g. general relativity.

# 4 DYNAMICS

## 4.1 Curved Spacetime

Can develop gravitation theory 3 ways:

- Coordinate Invariance and Tensors
- Field Theory of Spin 0,1,2 Fields

spin 0: no couple to light ; spin 1: repulsive (em)

spin 2: $T^{ab}\langle h_{ab}h_{a'b'}\rangle T^{a'b'}$ propagator

- Equivalence Principle / Grav Redshift / Differential Geometry

**FIGURE: Spacetime Diagram of Parallels**

Acceleration $\Rightarrow$ over time get $v = gh \Rightarrow z = v = gh$

But $t' \neq t_0 \Rightarrow$ parallel not parallel, i.e. curvature.

### 4.2 Einstein Field Equations

Each way leads to the same equations of General Relativity:

$$G_{ab}[g_{ab}] \equiv R_{ab} - (1/2)Rg_{ab} \qquad \text{(spacetime)}$$

$$R_{ab} - (1/2)Rg_{ab} = 8\pi T_{ab} \qquad \text{(Einstein Field Eqs)}$$

$$\text{(particle physics)} \qquad 8\pi T_{ab} \equiv (2/\sqrt{-g})[\delta(\sqrt{-g}\mathcal{L})/\delta g^{ab}]$$

where $T_{ab}$ is the energy-momentum tensor and $\mathcal{L}$ is the Lagrangian.

### 4.3 Perfect Fluids

GR says all forms of energy have mass: energy, momentum, pressure.

$$G_{ab}(a,k) = 8\pi T_{ab}(\rho, p)$$

### 4.4 Newtonian Approach

*Our universe is remarkably simple enough to treat quasiNewtonian.*