Power Spectrum of Perturbations

The power spectrum of density perturbations $\delta \equiv (\delta\rho/\rho)$, with index n, is

$$P_k \equiv |\delta_k|^2 \propto k^n.$$

a) Derive the expression for the rms density fluctuations on a comoving scale λ

$$\left(\frac{\delta\rho}{\rho}\right)_\lambda \sim k^{3/2}|\delta_k| \propto \lambda^p$$

and find the index p in terms of n.

b) Find the index m for the rms mass fluctuations on a comoving scale λ:

$$(\delta M/M)_\lambda \propto M^m.$$

What is the criterion on n such that structures grow in a top-down manner (large masses collapse first)? In a bottom-up manner? If the primordial spectrum has $n = 1$ and the CDM transfer function goes as $T(k) \sim k^{-2}$ due to the horizon entry effect, what m does this correspond to and what is the physical interpretation of the mass fluctuation spectrum?

c) The dynamics of the perturbations are driven by the fluctuations in the gravitational potential ϕ. Find the index f for the rms potential fluctuations

$$\delta\phi_\lambda \propto \lambda^f.$$

What is the physical significance of the scale invariant power spectrum index $n = 1$? Also explain the difference between a process being scale free and being scale invariant. Discuss which, if either, is Newtonian gravity.