Nucleosynthesis and T_{CMB}

A state of the early universe hot enough to have primordial nucleosynthesis would leave evidence behind in the form of a thermal glow, i.e. a cosmic radiation background at some temperature T_0 today. This prediction was made by Gamow in 1948 and refined by him, Alpher, and Herman but then forgotten until resurrected under different circumstances by Dicke and Peebles in 1965.

Reproduce a simple version of Gamow's argument as follows to find T_0. In order to produce helium the particle energies must be high enough to fuse deuterium but low enough not to dissociate deuterium (binding energy 2.2 MeV). Since this is of order 1 MeV treat the universe as radiation dominated. One could then set $\Gamma = H$ to find T_{ns}, as in lecture. Suppose one takes $T_{ns} = 1$ MeV. To find T_0 one just cools the background temperature by the expansion factor: $T_0 = T_{ns}/z_{ns}$, so one needs to determine z_{ns}.

a) Try deriving T_0 by finding z_{ns} from t_{ns} and t_0. From dating you know $t_0 = 10^{10}$ y. Derive t_{ns} from $H(T_{ns})$. First do this assuming the universe stays radiation dominated, and then repeat the calculation assuming it made a transition to matter domination at some $z_{eq}(T_0)$, using $\Omega_m \approx 1$. Do you expect T_0 to be less or greater in the second case?

b) Now derive T_0 by finding z_{ns} from n_{ns} and n_0. What happens if you use the number density of photons? It is better to use the number density of baryons; from the deuterium freezeout condition $n(T_d = 10^9\, K) = 10^{20}\, cm^{-3}$.