# Freezeout and Recombination

Early universe was ferment of particle interactions.

Was in **thermal equilibrium** – reaction rates equal forward and backward.

Expansion of universe gave **Hubble dilution**

- diluted particle density: $n \sim a^{-3}$
- reduced particle energies: $T \sim a^{-1}$

Reaction rate equation for comoving number density

$$\dot{n} = S(t) - n\,\Gamma(t)$$

$$\text{source } S(t) \quad ; \quad \text{sink (interaction) } \Gamma(t)$$

Mostly 2-body reactions because dilute:

$$n_N(t = 10^2 s) = 10^{20}\mathrm{cm}^{-3} \ll n_N(\mathrm{star\,core}) = 10^{29}\mathrm{cm}^{-3}$$

- **Quasistatic Equilibrium**:

$$n = S(t)/\Gamma(t), \qquad \Gamma \gg H$$

- Dilution:

  $n$ falls below equilibrium

- **Freezeout**:

If interactions rapid then quick transition from equilibrium to no reactions

$$\begin{aligned} N_{int}(> t) &= \int_t^\infty dt'\,\Gamma(t') \\ &= (n-2)^{-1}(\Gamma/H)_t \end{aligned}$$

$$\Gamma \sim T^n \qquad\qquad T \sim a^{-1} \sim t^{-1/2} \qquad\qquad H = (1/2)t^{-1}$$

So freezeout condition is $\Gamma < H$. Freezeout time $t_f$ with

  - $N_{int}(> t_f) < 1$
  - fixed point solution $n(\infty) \approx S(t_f)/\Gamma(t_f)$

**Interactions**

  Interaction rate $\Gamma = n\,\langle \sigma v \rangle$

- *Weak Interactions*

Carried by $W$, $Z$ bosons (massive) $\quad : \quad \sigma \sim G_F^2 T^2$

$$n + \nu \leftrightarrow p + e^-$$

$$n \sim a^{-3} \sim T^3 \qquad ; \qquad \sigma \sim T^2 \qquad ; \qquad v \sim 1$$

$$\Gamma \sim T^5 \qquad ; \qquad H \sim T^2 \qquad \Rightarrow \qquad \Gamma/H \sim T^3$$

$\Rightarrow$ In equilibrium in early universe, Frozen out today

Nucleosynthesis of light elements: abundances fixed primordially

- *Generic Interaction*

Weaker Interactions $\longrightarrow$ Earlier Decoupling ($\Gamma < H$ earlier)

Earlier Decoupling $\longrightarrow$ Lower Abundance and Temperature

From **equipartition principle**: energy shared among all particle **species**.

**Degrees of freedom $g_*$**

$$\rho = (\pi^2/30)\, g_*\, T^4$$

$$g_* = \sum_{\text{bosons}} g_i\, (T_i/T)^4 + (7/8) \sum_{\text{fermions}} g_i\, (T_i/T)^4$$

Comes from particle statistics

$$\rho_i(T_i) = (2\pi)^{-3} g_i \int d^3p\, f(p)\, E(p)$$

since phase space function $f = d^3n/d^3p = (e^{E/T} \pm 1)^{-1}$.

Relativistic $\to$ $E = p$.

- *Fermions*: half integral spin, e.g. neutrinos, electrons, nucleons.

- *Bosons*: integral spin, e.g. photons.

$$\rho_i \sim g_i \int dx \frac{x^3}{e^{x/T} \pm 1}$$

Direct evaluation or trick:

$$\frac{1}{e^x+1} = \frac{1}{e^x-1}\left(\frac{e^x-1}{e^x+1}\right) = \frac{1}{e^x-1}\left[1-\frac{2}{e^x+1}\right] = \frac{1}{e^x-1} - \frac{2}{e^{2x}-1}$$

$$\begin{aligned}\rho_{i,f} &\sim g_i\left[\int dx\frac{x^3}{e^x-1} - 2\cdot\frac{1}{16}\int d(2x)\frac{(2x)^3}{e^{2x}-1}\right] \\ &= \frac{7}{8}\,\rho_{i,b} = \frac{7}{8}\cdot\frac{\pi^2}{30}g_i T_i^4\end{aligned}$$

**Entropy**

Expansion adiabatic so entropy conserved: $S \sim \rho V/T$ by First Law. So

$$g_{*s}T^3a^3 = \text{constant}$$

where $g_{*s}$ is $g_*$ with $T^3$ not $T^4$.

$T \sim a^{-1}$ *only* when $g_{*s}$ constant: no species uncouple.

EXAMPLE: *Neutrino Decoupling*

Neutrinos decouple at $t_d$ so $T_\nu = T_d\,(a_d/a)$.

But photons still coupled to $e^-, e^+$ which annihilate, so photons heated.

$$g_* = g_{*s} = (2)_\gamma + \frac{7}{8}\cdot[(2)_{e^-} + (2)_{e^+}] = 5.5 \qquad \text{(before)}$$

$$g_* = g_{*s} = (2)_\gamma \qquad \text{(after)}$$

So photons heated (actually just don't cool as much), $T \sim a^{-1} g_{*s}^{-1/3}$,

$$T_\gamma/T_\nu = (5.5/2)^{1/3} = (11/4)^{1/3} = 1.40 = \frac{2.73\,K}{1.95\,K}$$

**Entropy Density** and **Specific Entropy**

Entropy density

$$s = S/V = (2\pi^2/45)\,g_{*s}T^3 = 1.80\,g_{*s}n_\gamma$$

Specific entropy is entropy per baryon.

Often written as **baryon-photon ratio**.

$$\eta \equiv n_b/n_\gamma = 7.04\,(s/n_b)^{-1} = 2.7\times 10^{-8}\,\Omega_b h^2$$

- constant since baryon-antibaryon annihilation
- high entropy puzzle: $(s/n_b)_U \approx 10^{10} k_B$ but $(s/n_b)_\star \approx k_B$

related to matter-antimatter asymmetry, or baryogenesis, puzzle

- important for nucleosynthesis and electromagnetic interaction freezeout

EXAMPLE: *Electron-Positron Freezeout* ($e^- e^+ \leftrightarrow \gamma\gamma$)

Suppose cross section $\sigma = r_e^2$. Then $\Gamma = n_e r_e^2$.

$$n_e \approx n_\gamma = n_{\gamma,0}\,(1+z)^3 = n_{\gamma,0}(1\,MeV/T_0)^3(T/1\,MeV)^3$$

The Hubble parameter is

$$H = 1/2t = (1/2)(T/1\,MeV)^2(1\,s)^{-1}$$

Freezeout condition

$$\Gamma/H = 2n_{\gamma,0}(1\,MeV/T_0)^3 r_e^2 c(T/1\,MeV)$$

$$= 1.5\times 10^{17}(T/1\,MeV)$$

Strongly in equilibrium at $1\,MeV$.

- Energy independent cross section not realistic.

Try $\sigma = r_e^2\, e^{-1\,MeV/T}$ below $1\,MeV$.

Represents difficulty in creating electron-positron pairs below threshold.

- Velocities go nonrelativistic.

Try $v = (T/1\,MeV)^{1/2}$ below $1\,MeV$.

Freezeout solution: $\Gamma/H = 1$ at $T = (1/35)\,MeV$, $z = 10^8$.

EXAMPLE: *Hydrogen Recombination* $(p + e \to H + \gamma)$

Ionization energy of hydrogen atom 13.6 eV $(z = 40,000)$.

But photon-baryon ratio $\eta^{-1}$ so high that even few photons on Wien tail of blackbody enough to ionize.

Analogy of ocean-land interactions. Heat capacity of water 5 times earth so ocean moderates land temperatures. Radiation $\eta^{-1}$ times matter so tight coupling: recombination immediately undone by ionization.

Approximate treatment by equilibrium **Saha equation**

Boltzmann formula for number density of species $i$

$$n_i = g_i (m_i T/2\pi)^{3/2} e^{-(m_i - \mu_i)/T},$$

Conservation of charge: $n_e = n_p$; chemical potential: $\mu_p + \mu_e = \mu_H$.

Define **ionization fraction** $X = n_e/n_b = n_e/(n_p + n_H)$.

$$X = \left(-1 + \sqrt{1+4S}\right)/2S,$$

$$S = [4\zeta(3)\sqrt{2/\pi}](T/m_e)^{3/2} \eta\, e^{Q/T},$$

Ground state binding energy $Q = m_p + m_e - m_H = 13.6$ eV. [] = 3.84.

Asymptotic behavior:

$$m_e \gg T \gg Q \to S \ll 1 \Rightarrow X \to 1 \qquad \text{(fully ionized)}$$

$$T \ll Q \to S \gg 1 \Rightarrow X \to S^{-1/2} \ll 1 \qquad \text{(recombined)}$$

Steep transition at $S \approx 1$. Doesn't occur at $T \approx Q$ because small prefactors. Since $T/m_e \approx \eta^{2/3}$,

$$T_{rec} \approx \frac{Q}{-2\ln\eta} = \frac{13.6\,eV}{40} = 0.3\,eV$$

$$T_{rec} = 0.3\,eV = 3600\,K \quad ; \quad z_{rec} = 1300 \quad ; \quad t = 2 \times 10^5\,\mathrm{y} = 6 \times 10^{12}\,s$$

High entropy of universe (low baryon-photon ratio $\eta \approx 3 \times 10^{-10}$) prevents recombination near binding energy. Stars have $\eta \approx 1$ so recombination at binding energy.