Growth of Density Perturbations

For density perturbations to the matter with wavelengths much greater than the Jeans length, the time evolution is given by

$$\ddot{\delta} + 2H\dot{\delta} - (3/2)\Omega_o(t)\, H^2\, \delta = 0.$$

a) Rewrite this equation with the dependent variable being the scale factor a. Write any derivatives of a or H in terms of H and the deceleration parameter q.

b) Consider the case of a critical matter universe where, on the scales considered, only a constant fraction Ω_{cl} clumps to form structure. (Possible realizations are a cold + hot dark matter universe or a dark + baryonic matter universe). In this case the Ω_o in the source term of the evolution equation is replaced by Ω_{cl}. Solve the equation to find the behavior of the growing mode: $\delta \propto a^m$. Interpret. Check the limits $\Omega_{cl} = 0$ and 1.

c) Consider the case of an open universe at a time dominated by the curvature. Write the evolution equation, substituting in for q and $\Omega_o(t)$, keeping only the leading order for the coefficient of each term. What happens to the source term for the growth as the universe expands? Try a solution $\delta \propto a^m$ as a gets large. Find the dominant mode and give the physical interpretation (include explanation of the roles of both the drag and source terms).