Multi-Color Light Curve Measurements

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1. Introduction

Empirical studies of multi-color light curves have proven invaluable for the use of Type Ia supernovae (SNe Ia) as distance indicators. Corrections based on light curve shapes of Branch-normal supernovae have reduced the dispersion in the peak magnitudes down to ~ 0.15 mag. Color information provides an additional correction for dust extinction and/or an intrinsic second parameter that further reduces the dispersion down to the 10% level, the level of experimental uncertainty.

It comes as no surprise that broad-band photometry provides a dataset with which we can study possible SN Ia heterogeneity. Photometric data are the highest signal-to-noise measurements available and are (relatively) easily obtained from the local Hubble flow out to cosmological distances. As described elsewhere in this document, different properties of the progenitor system are expected to manifest themselves as variations in the brightness and shapes of the ensuing supernova light curve.

In this note, we describe how those parameters are derived from the light curve, how the error on each one transforms into an added statistical error on the final magnitude, and the requirements which have to be fulfilled in order to bound possible evolutionary effects.

2. Light-curve Fit Parameters

2.1. The Light-Curve Template

Type Ia supernovae have been observed to be very homogeneous and describable by a single light-curve template (1). However, a small heterogeneity in light curve shapes that can be viewed as a stretching of the light-curve time-axis (2; 3) has been seen and in fact correlates with the supernova peak absolute magnitude. Theories predict variations in rise time ($t_{max} - t_{exp}$) and plateau level ($M_{peak} - M_{tail}$) with shifts in intrinsic supernova brightness. These effects have not yet been observed in the present data because of the limitations of their quality. Some studies (4; 5) have shown that a parametrised "composite light curve" fit to photometric data can yield interesting statistics such as stretch, rise time and plateau level. In the present study, we will use parametrised templates for rest frame B and V-band light curves. These templates are described by three sequential periods in the supernova's light curve (see Figure 1) :

$$Flux = \begin{cases} a(t-t_{exp})^2 & \text{for } t < t_{join} = -12 \text{ days} \\ F_{max}\mathcal{T}\ (s(t-t_{max})) & \text{for } t_{join} < t < 30 \text{ days} \\ \mathcal{T}\ ((t-t_{max}))10^{-0.4[(t-t_{max})/\tau+b]} & \text{for } t > 30 \text{ days.} \end{cases} \tag{1}$$

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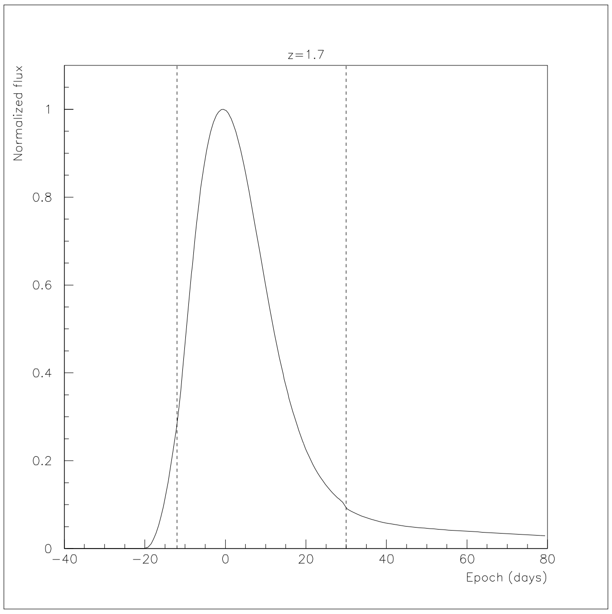

Fig. 1.— An example of a composite light curve, fit to supernova data. The point of inflexion between the parabolic rise and the central region is located near $t_{join} = -12$ days. Then the modified Leibundgut template extends until $t \simeq +30$ days. It is followed by the "plateau region", fit to an exponential decay function.

- The first period describes the early development of the explosion, with a parabolic rise in flux, starting at the explosion time t_{exp} and extending until $t_{join} \simeq t_{max}$ - 12 days, where t_{max} is the day of maximum.

- The second period contains the "Extended Leibundgut" template, $\mathcal{T}$, generated from a B-band (V-band) template in magnitude, converted and normalised to unity at the peak in flux (1). This part describes the core of the explosion, from t_{join}, up to the maximum at t_{max}, and down to the plateau region. Each supernova is characterised by its maximum flux, F_{max} and "stretch factor" s. By definition, the template has $F_{max} = 1$ and $s = 1$.

- The last period is the plateau region which starts after $t_{max} + 30$ days and corresponds to the radioactive decay of ^{60}Co. The slope τ is expected to be equal to the time constant of ^{60}Co and b represents the "plateau" level. The stretch is not included in the template $\mathcal{T}$ as it is not applicable in the plateau. This has the benefit of decoupling parameters describing the middle and late-time behaviour of the light curve. No continuity is required for the moment as we are developing a more robust model for this part of the light curve (freedom in b does allow for continuity). Eventually, a continuity condition will be implemented.

The template has 8 parameters with one continuity condition : we require the light curve to be continuous at t_{join}. The parameters t_{exp}, F_{max}, t_{max}, s, b, τ and color $(B - V)$ are derived for each supernova from a simultaneous fit of the B and V band light curves to the composite templates. The two filters share the same stretch and time of explosion and have a fixed relative times of maximum. The plateau and rise time are fit only in B. All fits are performed in flux, where the errors are Gaussian and symmetric.

2.2. Fitting Procedure

Simulated SNAP light curves are fit to the composite template defined in the previous section to determine the light-curve parameters and their errors. Each point on an individual light curve has the same exposure time; we generate and fit curves with different exposure times to explore the relation between photometric precision and light-curve parameter determination. The cadence of observations is roughly every four days in the supernova frame.

The fitting procedure consists of minimising

$$\chi^2 = \sum_{ij} (F_i - F_i^{exp})^T . \sigma_{ij}^{-1} . (F_j - F_j^{exp}) \tag{2}$$

where σ_{ij} is the experimental covariance matrix provided by the light-curve simulator. The minimisation is performed using Minuit. Since there is no Monte-Carlo smearing the final value of each parameter is equal (within rounding errors) to its input value. Minuit provides the covariance matrix C_{ij} of the fit parameters, which gives a measure of the parameter uncertainties which in turn can be used to compute the final corrected magnitude with exact correlations.

3. Error on the Corrected Magnitude

The peak magnitude is related to the parameters according to the following model:

$$\begin{aligned} M_B = M_{peak} + \alpha(s-1) + K_{corr} + RE(B-V) + R^{'}[(M_{peak} - M_{tail}) - (M_{peak} - M_{tail})_0] \\ + R^{''}[(t_{max} - t_{exp}) - (t_{max} - t_{exp})_0]. \end{aligned} \tag{3}$$

where

$$M_{peak} = -2.5 \log F_{max} + \text{zeropoint}. \tag{4}$$

The values for the coefficients entering correction terms in the magnitude are those presented in Table 1. They will eventually be deduced empirically from ongoing nearby supernova searches. Currently only stretch and the color corrections are well enough understood to apply corrections in this manner (2; 3; 5; 6). The present precision is not sufficient to deduce the $R^{'}$ and $R^{''}$ coefficients, and theoretical models have been used (7) to provide their order-of-magnitude expectations.

When using Type Ia supernova peak magnitudes to measure distances, we can use light-curve parameters to either split the supernovae into subsamples to compare like to like, or we use them to directly apply magnitude corrections. In the former case the subsamples are to be selected using these parameters, with bin sizes that encompass the targeted magnitude errors. Table 1 lists the observed (or expected) relationships between the light-curve parameters and the peak intrinsic magnitude.

Table 1: Coefficients for the various corrections to the fit magnitude

Parameter	$\partial M/\partial a_i$	Comment
α	1.9	Stretch
R	4.1	Host galaxy extinction
$R^{'}$	0.25	Peak - to - tail
$R^{''}$	0.1	Rise time
K_{corr}	-	Added directly

When directly applying magnitude corrections, the covariance matrix between all fit parameters as given by the fit is used to construct the statistical error on the magnitude as:

$$\sigma_M^2 = \sum_{ij}[\partial M/\partial a_i]^T.[Cij].[\partial M/\partial a_j], \tag{5}$$

where $\{a_i\}$ are the fit parameters and the partial derivatives are taken from Table 1.

4. Experimental Requirements

In designing an experiment that corrects for supernova peak magnitude heterogeneity, either using subsamples or corrections, we need to set a target magnitude error that propagates into parameter error requirements. Parameters that give random residuals in magnitude after correction have errors that can be reduced by large number statistics. This is the case for stretch and extinction corrections. For these observables we demand an individual error contribution less than the intrinsic supernova magnitude dispersion after correction, < 0.1 mag. For parameters with as-of-yet unknown behaviour and for which it is not clear that a correction can be applied, we here consider a target 0.02 mag error bound. We first examine each parameter separately and then consider the total expected error on magnitude when corrections are applied. These are translated into signal-to-noise requirements on the light curve which can be used to determine exposure times as function of redshift.

4.1. Requirement on Each Fit Parameter

Figure 2 shows directly the errors of each parameter in the fit. In this simplified approach, the error on each parameter propagates into the corrected magnitude via the coefficient $\partial M/\partial a_i$ from Table 1. Figure 3 shows the dependence of the contribution to the error on magnitude related to each term using simulated supernovae at redshift $z = 1$. At differing redshifts and supernova epochs, the error contributions from source, zodiac, and detector vary. However, the relative signal-to-noises of points on a light curve will not change greatly for the redshift ranges and exposure times considered here so we take the $z = 1$ case as a representative example.

We examine each of the fit components to determine which give the strongest constraints:

- Frequency of observation and rise time.

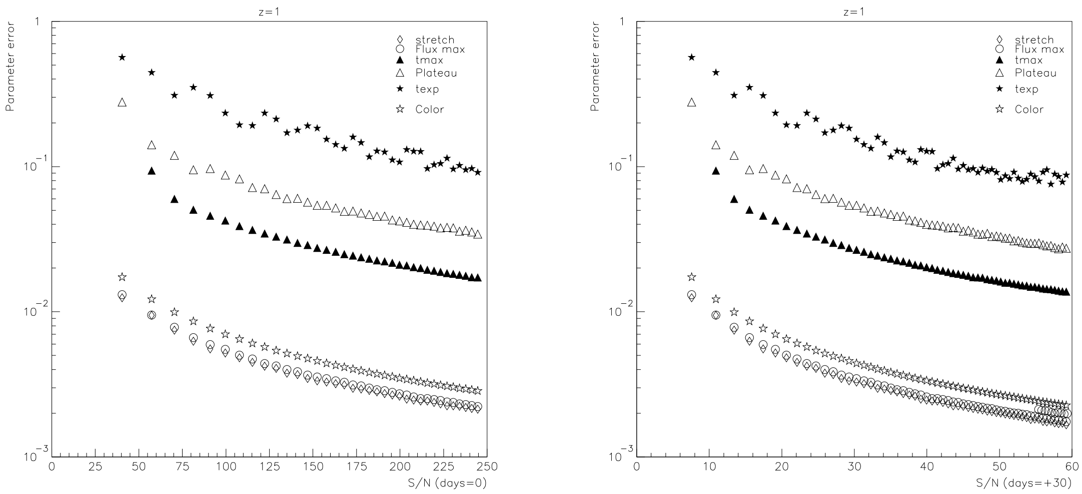

Fig. 2.— Direct errors on each fit parameter as a function of signal-to-noise at peak (left) and 30 days after peak (right) in the SN rest frame. The dominant error is related to the explosion time, followed by the plateau.

Potential systematics related to the progenitor make-up are connected to the rise time, $t_{max} - t_{exp}$. This region of the light curve extends typically between 12–18 days before maximum. Fitting a parabola on less than 3 points is not optimal, even with only one parameter. Therefore it seems reasonable to observe each sky field every 4 days in the supernova rest frame, which also guarantees early detection within 2 days of the explosion.

The rise time $t_{max} - t_{exp}$ can also be used as an indicator of opacity, fused ^{56}Ni mass, and potential differences in the ^{56}Ni distribution. It is estimated to be related to the magnitude by a coefficient $R^{''}$, of order 0.1. In this case, as shown on Fig.3, the contribution in the final error is small. But this correction needs to be studied when more data will be available.

- Plateau level.

  The level of the exponential decay at $t > t_{max} + 30$ days is an estimator of the mass of ^{60}Co in the terminal phase, which is highly correlated to the C/O ratio in the progenitor system, itself correlated with the peak magnitude. Therefore, a correction to the peak magnitude, proportional to the difference between peak and tail magnitudes will not only reduce the intrinsic dispersion, but will also enable us to control the systematic errors related to evolution. In practice, the error will depend on our level of understanding of the relation between magnitude, peak-to-tail ratio and redshift. Present theoretical work favours a slope $\partial M/\partial a$ of order 0.25. This appears on Figure 3 to be one of the largest contributions to the error. In the present calculation, no continuity with the other part of the template is required and no constraint is applied on the slope; a continuity condition or bounding values on the slope should improve the plateau errors.

- Peak flux

  The flux at maximum F_{max} is determined by the observations during some 10 days, when the supernova is at its maximum brightness. This means that the peak flux results mainly from 3

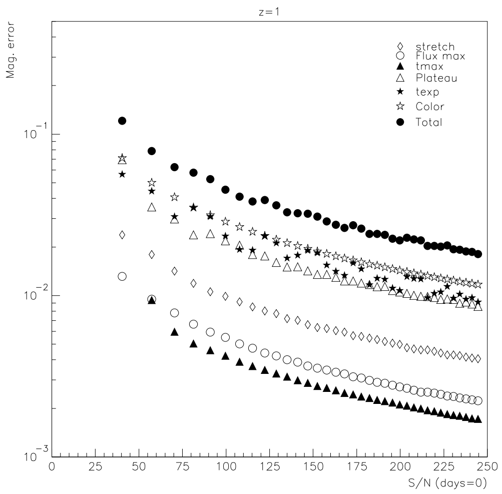

Fig. 3.— Individual components to the magnitude error from each parameter in the fit as a function of the signal-to-noise at peak (left) and 30 days after peak (right) in the SN rest frame. Each component is obtained by multiplying the error on the parameter by the slope $\partial M/\partial a_i$. The black filled circles display the total combined error on the magnitude which includes the covariance in the fit parameters. The dominant contribution comes from the color correction, followed by the plateau.

observations at $t \simeq t_{max}$. In any case, the observational requirement is less stringent than the one related to the color measurement for host galaxy extinction, which is weighted by a factor 4.

- Host galaxy "normal" dust extinction

  Dust in the host galaxy causes absorption of the light emitted by the supernova. As an extinction indicator we use the color excess $(B-V)$ as one of the parameters in the fit. Its error is approximately $\sqrt{2}$ times the error on the peak magnitude in one color assuming that the B and V magnitude errors are comparable. The extinction coefficient of diffuse Milky Way dust is of order 4.1 which makes this correction the most sensitive parameters of the fit.

- Stretch

  The stretch correction takes the form $\alpha(s-1)$ with α estimated from a two parameter fit (stretch and color) on current data to be 1.9. This correction is less stringent than the ones from color or plateau.

4.2. Requirement on Parameter Binsizes

Considering each parameter individually, we estimate the binsizes needed for a subsample study of supernovae. To distinguish a 0.1 mag bin in extinction correction requires an exposure time that gives $S/N \sim 5$ at 30 days after peak; the requirement for stretch is much less demanding. Distinguishing 0.02 magnitude correction bins for the other parameters is more demanding. The date of explosion and plateau measurements require an exposure time that gives $S/N \sim 20$ at 30 days after maximum.

4.3. Requirement on the Combined error

We can determine how light-curve measurement error propagates into a final corrected peak-magnitude uncertainty assuming that all these parameter corrections are found to be independent and unbiased.

The total error on magnitude is computed using equation 5. The full covariance matrix is used, as well as the derivatives from Table 1. For a complete treatment of the statistical errors, the contribution due to K corrections should be added in quadrature. The main goal is to keep the total error well below the intrinsic dispersion which is estimated, after all corrections, to be around 0.1 mag. Figure 3 shows that this constraint on the correction error requires a signal-to-noise of at least 50 at peak and 10 after 30 days in the rest frame.

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