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DARK ENERGY AND THE CMB

Martin White
Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138

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ABSTRACT

The CMB anisotropy constrains a wealth of cosmological parameters. Of direct relevance to the dark energy, the primary constraints are on the distance to the last scattering surface and the physical matter density $\omega_{\rm mat}$. The evolution of the equation of state, w, has little effect on the CMB which constrains a weighted measure $w_{\rm eff}$. If $w_{\rm eff} \approx -1$ it is next to impossible to determine the nature of the dark energy through anisotropy measurements alone. With *MAP* and *Planck* it should be possible to measure $w_{\rm eff}$ to between 10-30% accuracy, with the primary degeneracy being with $\Omega_{\rm mat}$ if the curvature is assumed known. To break this degeneracy or to begin to probe the evolution of the equation of state (e.g. dark energy potential if it is a rolling scalar field) requires measurements beyond the anisotropy. In addition to the SNe, the most promising of these appears to be the evolution of clustering, as probed for example by the counts of rich clusters of galaxies.

Subject headings: cosmology: theory – dark energy

1. INTRODUCTION

While cosmology has made tremendous progress in the last few years, major mysteries remain. Perhaps the most vexing, and with the strongest implications for our understanding of fundamental physics, is the nature of the dark energy believed to be causing the accelerated expansion of the universe.

It is therefore interesting to ask what we could learn about the dark energy from a cosmological perspective. Since the dark energy is required to be smooth, except possibly near the horizon scale, all of its cosmological effects come in through its effect on the expansion rate $H(z)$. Specifically it alters the distance-redshift relations, cosmological volumes and the growth of perturbations, all of which are integrals of the inverse Hubble parameter over redshift.

In this note I briefly describe the constraints on the dark energy, X, which can come from studying the CMB anisotropy, and how these constraints complement those from other areas of astrophysics. The focus will be on models where the gravitational sector of the theory is unchanged (comparatively little work has been done on models with modified gravity). In this case we need to specify the stress-energy tensor of the dark energy, which can be a cosmological constant, or 'X-matter' (Turner & White 1997), for example a rolling scalar field [1] (Kodama & Sasaki 1984; Ratra & Peebles 1988a, 1988b; Coble, Dodelson & Frieman 1997; Ferreira & Joyce 1998; Caldwell, Dave & Steinhardt 1998).

Since several introductory reviews of CMB anisotropy physics exist already (White, Scott & Silk 1994; Scott & White 1995; Hu, Sugiyama & Silk 1997; Bennett, Turner & White 1997; Smoot & Scott 1997; Bond 1998; Kamionkowski & Kosowsky 1999; an annotated bibliography of more than 280 references can be found in White & Cohn 2001) we will not repeat that material here.

[1] In general there are 10 components to $T_{\mu\nu}$ and 4 constraints from energy-momentum conservation leaving us with 6 free functions describing X. These decompose under $SO(3)$ as 2 scalar, 2 vector and 2 tensor modes. The minimal assumption is that the vector and tensor modes are zero and that the anisotropic stress of the scalar modes also vanishes. This is satisfied by the scalar field models, where the remaining free function is the scalar field potential. These models require minimal modifications to the standard formalism and high precision in the calculations can be achieved.

2. WHAT THE CMB CONSTRAINS (WELL)

While important constraints on parameters and tests of models can come from the gross features of the anisotropy spectrum (as outlined for example in Hu & White 1996; White 2001) precise measurements of the parameters require fitting models to high precision data. In the modern era this typically means a variant of the highly successful class of cold dark matter models, with the anisotropy spectrum computed using a numerical Boltzmann code such as CMBFAST (Seljak & Zaldarriaga 1996). A word of caution, when interpreting cosmological parameter estimates it is important to realize that the results can depend strongly on what is held fixed, and for error forecasts on what the assumed fiducial model is. In particular, if degenerate directions exist it is important to pick a good parameter basis.

For the CMB the main parameters are those from our model of the early universe: the scalar and tensor normalizations, spectral indices and running of the spectral index; and those which come from cosmology: the physical matter, $\omega_{\rm mat} \equiv \Omega_{\rm mat} h^2$, and baryon, $\omega_B \equiv \Omega_B h^2$ densities[2] the density in dark energy, Ω_X, spatial curvature, Ω_K, the optical depth to last scattering, τ, and the effective equation of state of the dark energy $w = p/\rho$ if it is not a cosmological constant, etc. In this basis, the Hubble constant is a derived parameter $h(\omega_{\rm mat}, \Omega_K, \cdots)$ and so does not enter explicitly.

The CMB temperature and polarization anisotropy spectra have 3 regimes. At large scales we are probing primarily the initial conditions, with a slight sensitivity to late time effects such as decaying gravitational potentials. On degree scales we have the acoustic peaks, arising from

[2] We follow usual practice and denote the density of component i in units of the critical density $\rho_{\rm crit} \equiv 3H_0^2/(8\pi G) = 1.8788 \times 10^{-29}\, h^2\,{\rm g\,cm^{-3}}$, as Ω_i. The Hubble constant we parameterize as $H_0 = 100h\,{\rm km\,s^{-1}\,Mpc^{-1}}$.

oscillations in the tightly coupled baryon-photon fluid at last scattering. On scales of a few arcminutes we have the damping tail, which is almost independent of the source of the fluctuations. Most of the constraints on cosmological parameters come from the acoustic peak region, where the spectrum has a lot of structure encoding information about the conditions at last scattering.

To zeroth order therefore we can consider the CMB as being generated at last scattering ($z \simeq 1200$) and then projected onto the sky. The first step involves only the inputs and $\omega_{\rm mat}$ and ω_B, requiring higher order effects to break the degeneracy. The projection is controlled by the angular diameter distance to last scattering, r, through

$$\ell_{\rm feature} = k_{\rm feature} r \tag{1}$$

where the feature can be any of the acoustic peaks, the damping tail, the peak separation etc. The effects of Ω_X and w enter through r. In a flat universe $r = \eta_0 - \eta_*$, where η_* is the conformal time $[d\eta = dt/a(t)]$ at last scattering and η_0 is the conformal time today.

Thus high precision CMB experiments constrain well the physical densities of matter and baryons, $\omega_{\rm mat}$ and ω_B, and the angular diameter distance to last scattering, plus spectral slopes and amplitudes for the primordial spectrum. Constraints on other parameters are obtained from higher order effects and are therefore weaker. Of the higher order effects some important ones are: (a) the integrated Sachs-Wolfe effect (Sachs & Wolfe 1967), which arises because the blue-shift of a photon entering a decaying potential is not compensated by the exit redshift if the transit time is comparable to the decay time. This effect operates primarily on large scales and encodes a sensitivity to the energy contents at low redshift. (b) gravitational lensing, which arises because structure along the line of sight to the last scattering surface gives rise to gravitational potentials which deflect photons. This tends to smear out the anisotropy spectrum increasingly at smaller angular scales (Seljak 1996). (c) fluctuations in the dark energy itself (Caldwell et al. 1998). For currently preferred values of Ω_X the fluctuations in the field are only relevant for $\ell < 10$, and even there are a small effect. For $w < -0.6$ verifying that the dark energy arises from a particular model, e.g. a rolling scalar field, from CMB measurements will be next to impossible (Hu et al. 1999; hereafter HETW).

Two models which have the same input spectrum, the same $\omega_{\rm mat}$ and ω_B and the same comoving distance to last scattering will predict almost identical CMB spectra as shown in Fig. 1. The spectra differ only at very low-ℓ due to the integrated Sachs-Wolfe effect and at very high-ℓ due to gravitational lensing. For this reason we can parameterize most models of the dark energy by the amount of dark energy today, Ω_X, and an effective (constant) equation of state $w_{\rm eff}$. The latter is chosen to produce the same angular diameter distance to last scattering as the "full" model with a possibly time varying equation of state. If the equation of state is slowly and monotonically varying then (Huey et al. 1999)

$$w_{\rm eff} \simeq \frac{\int dz\ \Omega_X(z) w(z)}{\int dz\ \Omega_X(z)} \quad . \tag{2}$$

Thus the CMB provides essentially one direct constraint on dark energy, the angular diameter distance to last scattering (see Fig. 2), which depends on the matter density, the dark energy density and its evolution. Note that we have only one constraint here – the CMB cannot simultaneously be used to claim that the universe is flat *and* constrain $w_{\rm eff}$. To make progress we must *assume* that the

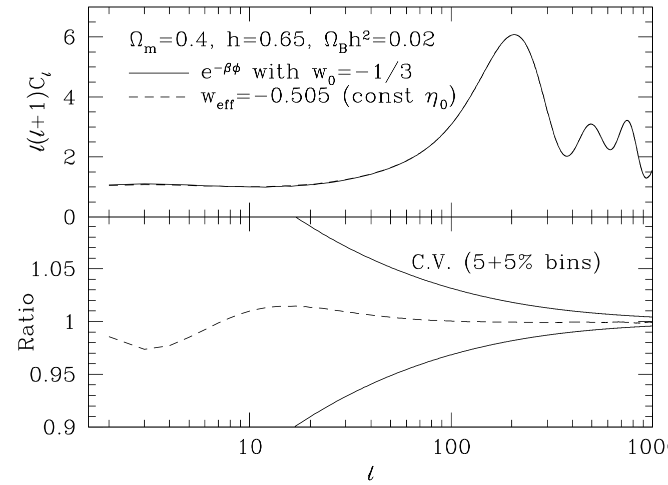

Fig. 1.— The effective equation of state and in the CMB. The upper panel shows the angular power spectrum for two scalar field models of the dark energy, the first with a time varying equation of state corresponding to an exponential potential $V(\phi) \propto \exp[-\beta\phi]$ and the second with a constant equation of state which gives the same conformal age of the universe. The bottom panel shows the ratio and the irreducible error bars, averaged in bins of $\Delta\ell = 5 + 0.05\ell$, due to cosmic variance.

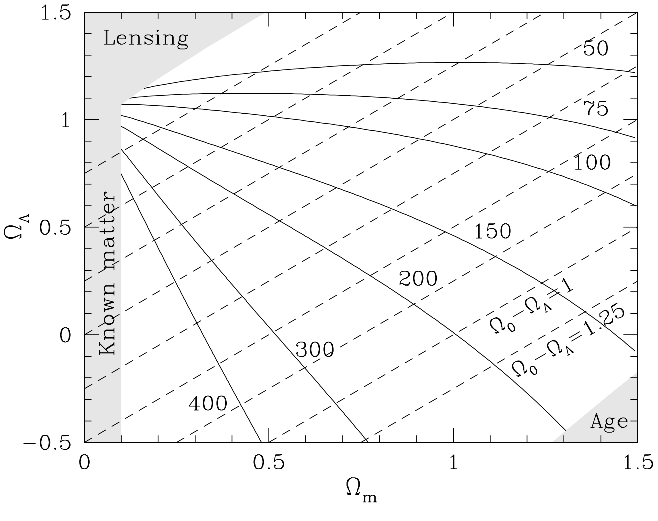

Fig. 2.— The first 'complementarity' plot (White 1998). Solid contours show the location of the first acoustic peak, in ℓ, as a function of $\Omega_{\rm mat}$ and Ω_Λ. The dashed lines are lines of constant $\Omega_{\rm mat} - \Omega_\Lambda$ which is approximately the combination probed by SN-Ia at intermediate redshift.

universe is flat. Unfortunately, without prior knowledge of $\Omega_{\rm mat}$, even for a flat universe there is a degeneracy in the between $w_{\rm eff}$ and $\Omega_{\rm mat}$ in the distance (see Fig. 3). However, the CMB will additionally constrain $\omega_{\rm mat}$ and ω_B quite strongly – so an accurate measurement of h would allow a determination of $\Omega_{\rm mat}$. It appears that traditional astrophysical methods are unlikely to precisely constrain h^2 in the near future.

3. COMPLEMENTARITY

Since it is weighted to such high redshift, where the dark energy density is negligible, the CMB provides relatively weak constraints on its nature. The real power of the CMB comes from combining its constraints with other complementary measures. We show one of several examples of this in Fig. 4, from HETW, which indicates how the constraints from a series of hypothetical experiments would constrain a model where the universe is assumed flat and the equation of state is a constant.

As is well known, the combination of parameters fixed by the distance to a certain redshift changes with the redshift. So a low-z measurement such as a SN survey and a high-z measurement such as the CMB provide complementary constraints.

The ellipses marked SDSS indicate the constraints from an artificial Sloan Digital Sky Survey. One of the advantages of such a survey is that is can provide a way to estimate h, which as discussed above provides significant extra knowledge to break degeneracies. How this is done is indicated in Fig. 5. The key is to be able to measure features in the matter power spectrum, such as the 'baryon bumps' (e.g. Eisenstein et al. 1998; Meiksin, White & Peacock 1999; Percival et al. 2001; Miller, Nichol & Batuski 2001), in redshift space. Non-linear effects make this measurement difficult at lower redshift, but a future high redshift galaxy survey may be able to constrain them very effectively (D. Eisenstein, private communication). On larger scales one could attempt to measure the roll-over in the matter power spectrum which is due to matter-radiation equality (Broadhurst & Jaffe 1999; Roukema & Mamon 2000, 2001; Cooray et al. 2001) though this is likely to be extremely difficult since the roll-over is quite broad.

Alternatively one could try to directly constrain $\Omega_{\rm mat}$. In addition to the distance-redshift relations we can use the growth of clustering, which being a competition between gravity and expansion, is sensitive to $\Omega_{\rm mat}$. It is desirable to probe the crucial redshift range $z \simeq 0-2$, where the dark energy begins to noticeably affect the expansion rate, with as much resolution in redshift as possible. Several authors, most recently Haiman, Mohr & Holder (2001), have suggested using the counts of clusters of galaxies to probe the evolution of the dark energy in this redshift range. Clusters are big, bright, sparse enough to cover large volumes and (relatively) theoretically tractable. While much work remains both theoretically and observationally to implement this approach, it holds great promise for a third independent constraint in the $\Omega-w$ plane which could be very tight.

4. CONCLUSIONS

The CMB anisotropy constrains a wealth of cosmological parameters. Of direct relevance to the dark energy, the primary constraints are on the distance to the last scattering surface and the physical matter density $\omega_{\rm mat}$. The evolution of the equation of state, w, has little effect on the CMB, which constrains a weighted measure $w_{\rm eff}$.

Thus for estimation of parameters from the CMB the

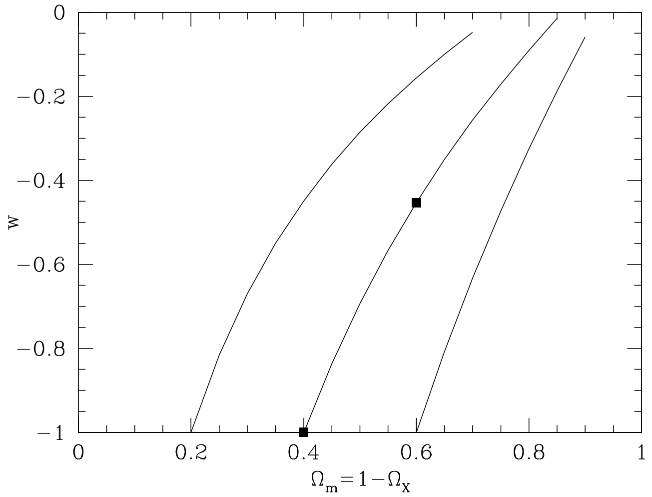

Fig. 3.— Three examples of lines of constant angular diameter distance to last scattering, illustrating the degeneracy between $\Omega_{\rm mat}$ and w at fixed curvature.

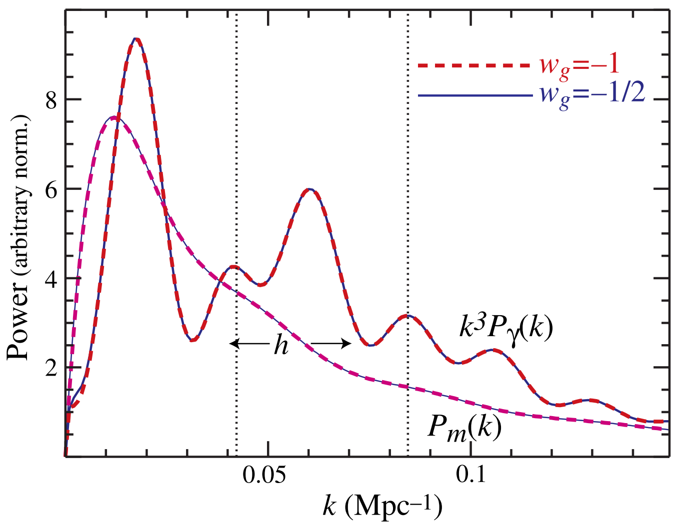

Fig. 5.— Measurement of h and the equation of state degeneracy. Acoustic features in both the CMB (k^3P_γ) and matter/galaxy power (P_m) spectra are frozen in at last scattering. Once the CMB acoustic peaks are calibrated in real space we can measure $\omega_{\rm mat}$ and ω_b. By sliding the galaxy power spectrum in redshift space (h Mpc^{-1}) until the features "match" determines h. This test is unaffected by late-time dynamics from the dark component. However, once h is determined, $\Omega_g = 1-\Omega_{\rm mat}$ follows from the CMB measurement of ω_m. The angular diameter distance measurement from the CMB then determines w.

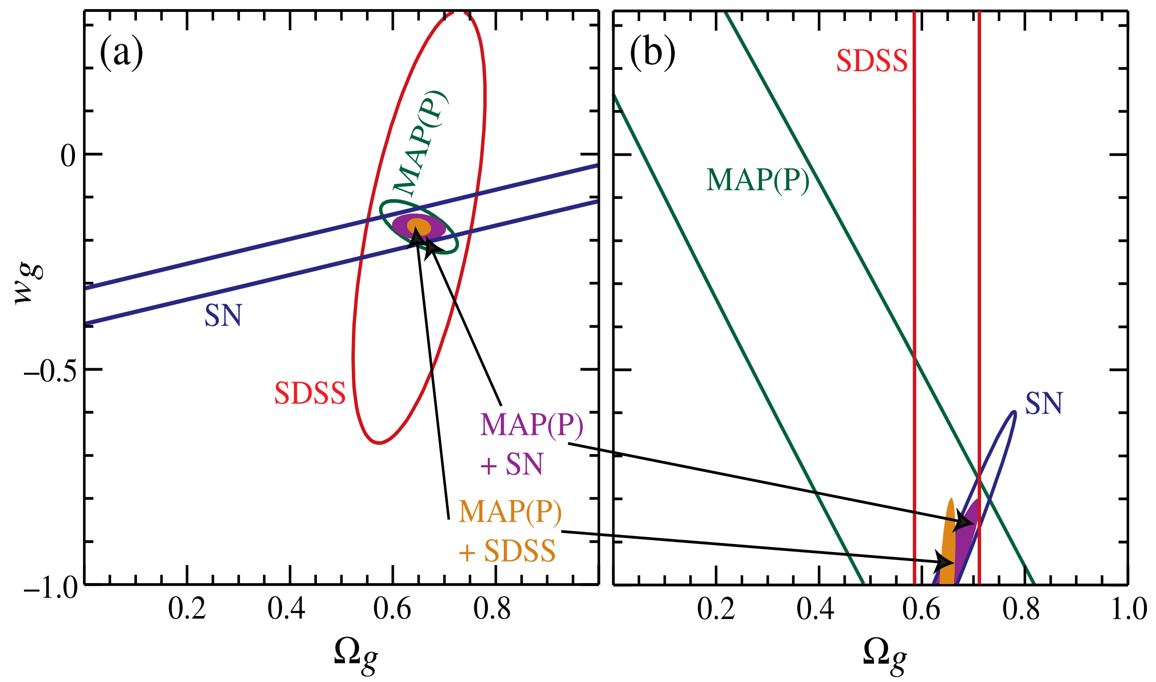

FIG. 4.— Forecast for the constraints in the $\Omega_g = 1 - \Omega_{\rm mat}$, w plane from CMB, SN and large-scale structure surveys (from HETW). The left and right panels show results for two fiducial models different in their assumed w.

dark energy adds one additional parameter: an effective, constant equation of state parameter $w_{\rm eff}$. Including this parameter affects the constraints that can be placed on Ω_X, Ω_K and h for example, but hardly affects early universe aspects such as tests of inflation. The fluctuations in the dark energy (e.g. speed of sound and viscosity) affect the CMB so little for $w_{\rm eff} \approx -1$ that it is next to impossible to determine the nature of the dark energy through anisotropy measurements alone.

With *MAP* and *Planck* it should be possible to measure $w_{\rm eff}$ to between 10-30% accuracy, with the primary degeneracy being with $\Omega_{\rm mat}$ if the curvature is assumed known. To break this degeneracy or to begin to probe the evolution of the equation of state (e.g. scalar potential if it is a rolling scalar field) requires measurements beyond the anisotropy. In addition to the SNe, the most promising of these appears to be the evolution of clustering, as probed for example by the counts of rich clusters of galaxies or features in the matter power spectrum.

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