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Lightcurve fits to nightly weighted-average data are not valid

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1. The problem

In most cases several SCP images for each object are obtained on each night it is observed, usually within less than 4 hours of one another. The large errors on these points make the plots look pretty ratty, and so weighted-average points are sometimes plotted for the sake of appearance. For example, `snmin02` produced `snmin02_ill_sn95126.2.ps` and `snmin02_sn95126.2.ps` for a two-color fit to sn95126. The first (snmin02_ill_*) is the "pretty" plot for further editing, for publication, and for talks. The second, four pages in this case, shows the actual fits to uncombined data. This can and must be examined by the experts.

2. Weighted-average point for a particular night

What about lightcurve fits to the weighted-average data? The validity of such fits is discussed every few years, most recently in private conversations at our SCP Collaboration Meeting. I couldn't demonstrate the invalidity of these fits on the spot, and hence this note.

Overbars are used to denote quantities for which this approach is used. All sums are over the points taken on the night under consideration. Also (pay attention!) we consider the contribution to the total χ^2 of just *one* night, and suppress subscripts for the rest of the nights to make the algebra more intelligible.

Let the weight of the jth point be $w_j = 1/\sigma_j^2$, and the intensity be y_j. For the weighted-average point $\overline{y}$ on the night being considered, we have

$$\overline{w} = \sum w_j \tag{2.1a}$$

$$\overline{y} = \frac{\sum w_j y_j}{\overline{w}} \ . \tag{2.1b}$$

The model intensity $f(t;\text{parameters})$ is nearly constant over the short time interval, so we take it as a constant for this night, on a given call to the subroutine `CHI` by `MINUIT`. The contribution of the average point to the total χ^2 is then

$$\overline{\chi}^2 = \overline{w}\,(\overline{y} - f)^2 \tag{2.2a}$$

$$= \overline{w}\,\overline{y}^2 - 2\overline{w}\,\overline{y} f + \overline{w} f^2 \ . \tag{2.2b}$$

3. Contribution to χ^2 from a night's uncombined intensities

In this (the usual) case,

$$\chi^2 = \sum w_j \left(y_j - f\right)^2 \tag{3.1a}$$
$$= \sum w_j y_j^2 - 2f \sum w_j y_j + \overline{w} f^2 \tag{3.1b}$$
$$= \sum w_j y_j^2 - 2\overline{w}\,\overline{y} f + \overline{w} f^2 \ . \tag{3.1c}$$

Since the last two terms of Eq. (2.2b) and Eq. (3.1c) are identical,

$$\overline{\chi}^2 \stackrel{?}{=} \chi^2 \tag{3.2a}$$

only if

$$\overline{w}\,\overline{y}^2 \stackrel{?}{=} \sum w_j y_j^2 \ . \tag{3.2b}$$

Unless I'm missing something, this is the case only if all the w_j's are the same. Although usually the exposure lengths are the same and hence the w_j's are nearly the same, there are exceptions. It is technically correct to do the lightcurve fits with the individual points and incorrect to fit to the weighted-average data.

Although we have stated that changes in $f(t; \text{parameters})$ over the short time interval are negligible, this is not technically correct either. The changes make a slight difference, especially if the data are obtained on a night during the initial rise. Again, it is better to fit to all of the actual data points rather than to the weighted average.